OCR
Oxford Cambridge and RSA

Monday 26 June 2017 – Afternoon

A2 GCE MATHEMATICS (MEI)

4756/01 Further Methods for Advanced Mathematics (FP2)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4756/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (54 marks)

1 **(a)** **(i)** By differentiating the equation $a\tan y = x$ show that

$$\int \frac{1}{a^2+x^2}\,\mathrm{d}x = \frac{1}{a}\arctan\left(\frac{x}{a}\right)+c\ .$$

[3]

The cartesian equation of an ellipse is $\frac{x^2}{4}+\frac{y^2}{9}=1$.

(ii) Show that the polar equation of the ellipse may be written in the form

$$r^2 = \frac{36\sec^2\theta}{9+4\tan^2\theta}\ .$$

[3]

(iii) By using the substitution $3u = 2\tan\theta$ show that the area enclosed by the ellipse and the lines $\theta = 0$ and $\theta = \frac{\pi}{4}$ is $3\arctan\left(\frac{2}{3}\right)$. **[7]**

(b) Obtain the first three terms of the Maclaurin series for $\mathrm{f}(x)$, where $\mathrm{f}(x) = \arctan(1+x)$. **[5]**

2 **(a)** The infinite series C and S are defined as follows.

$$C = -\frac{1}{2}\cos\theta + \frac{1}{4}\cos 2\theta - \frac{1}{8}\cos 3\theta + \ldots$$
$$S = -\frac{1}{2}\sin\theta + \frac{1}{4}\sin 2\theta - \frac{1}{8}\sin 3\theta + \ldots$$

By considering $C + \mathrm{j}S$, show that

$$S = \frac{-2\sin\theta}{5 + 4\cos\theta}.$$

Find a corresponding expression for C. **[9]**

(b) In an Argand diagram, O is the origin and points A and B are represented by the complex conjugate pair z_1 and z_2 respectively, where $0 < \arg z_1 < \frac{\pi}{2}$. The triangle OAB has side OA of length a.

(i) Show the above information on an Argand diagram. **[1]**

(ii) Show that $z_1 z_2$ is real, giving its value in terms of a. **[2]**

Triangle OAB is rotated anti-clockwise about the origin through γ radians, where $0 < \gamma < 2\pi$, and then enlarged through the origin with scale factor 3. The resulting new positions of A and B are represented by the complex numbers z_3 and z_4 respectively, where z_3 and z_4 form another complex conjugate pair.

(iii) State the value of γ. **[1]**

(iv) Find, in polar form (modulus-argument form), the complex number $\frac{z_3}{z_1}$. **[2]**

(v) Given that, in the original triangle OAB, AB also has length a, find the complex number $\frac{z_1}{z_4}$, giving your answer in the form $x + \mathrm{j}y$, where x and y are exact real numbers. **[3]**

3 **(a)** You are given the matrix $\mathbf{M} = \begin{pmatrix} k & 2 & 1 \\ 3 & -1 & 2 \\ 1 & 2 & -2 \end{pmatrix}$.

(i) Find the value of k for which $\mathbf{M}$ does not have an inverse. **[3]**

(ii) Find $\mathbf{M}^{-1}$ in terms of k. **[4]**

(b) The matrix $\mathbf{Q}$ is given by $\mathbf{Q} = \begin{pmatrix} 3 & 3 \\ 4 & 7 \end{pmatrix}$.

(i) Find the eigenvalues and corresponding eigenvectors of $\mathbf{Q}$. **[5]**

(ii) State a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{Q} = \mathbf{PDP}^{-1}$. **[2]**

(iii) Show that, for $n \geqslant 1$, $\mathbf{Q}^n = \frac{1}{8}\begin{pmatrix} 6 + 2\varphi & 3\varphi - 3 \\ 4\varphi - 4 & 6\varphi + 2 \end{pmatrix}$, where $\varphi = 9^n$. **[4]**

Section B (18 marks)

4 **(i)** Prove, from definitions involving exponentials, that $\operatorname{sech}^2 x + \tanh^2 x = 1$. **[4]**

(ii) Prove that

$$\operatorname{artanh} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right).$$

State the set of values of x for which this is valid. **[5]**

(iii) Solve the equation

$$3(\tanh^2 x - \operatorname{sech}^2 x) = \tanh x - 2\ ,$$

giving your answers in an exact logarithmic form. **[5]**

(iv) Find the exact value of

$$\int_{\operatorname{arsinh} 2}^{\operatorname{arsinh} 3} \frac{1}{\tanh x - \operatorname{sech} x}\,\mathrm{d}x\ .$$

[4]

END OF QUESTION PAPER

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Duration: 1 hour 30 minutes

Candidate forename		Candidate surname	

Centre number						Candidate number				

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Section A (54 Marks)

1 (a)(i)	
1 (a)(ii)	

1 (a)(iii)

1 (b)

2 (a)	
	(answer space continued on next page)

2 (a)	**(continued)**
2 (b)(i)	

2 (b)(ii)

2 (b)(iii)

2 (b)(iv)

2 (b)(v)	
3 (a)(i)	

3(a)(ii)	

3 (b)(i)

3 (b)(ii)	
3 (b)(iii)	

Section B (18 Marks)

4 (i)	

4 (ii)

4 (iii)	

4 (iv)

ADDITIONAL ANSWER SPACE

If additional space is required, you should use the following lined page(s). The question number(s) must be clearly shown in the margins.

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GCE

Mathematics (MEI)

Unit **4756**: Further Methods for Advanced Mathematics

Advanced GCE

Mark Scheme for June 2017

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✘	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
cwo	Correct working only
ww	Without working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the

establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation DM is used to indicate that a particular mark is dependent on an earlier [asterisked] mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance
1	(a)	(i)	$a\tan y = x$ $\Rightarrow a\sec^2 y\frac{\mathrm{d}y}{\mathrm{d}x} = 1$ $\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{a\sec^2 y} = \frac{1}{a\left(1+\frac{x^2}{a^2}\right)}$	M1	Differentiation attempted w.r.t. *x* or *y*. [$\sec^2 y$ seen]
			$= \frac{a}{a^2+x^2}$	A1	Correct simplified expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of *x*.
			so $\int \frac{1}{a^2+x^2}\mathrm{d}x = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + c$	A1	**AG** Completion to given answer www Omission of + *c* is A0
				3	
1	(a)	(ii)	$\frac{x^2}{4} + \frac{y^2}{9} = 1$		
			$\frac{(r\cos\theta)^2}{4} + \frac{(r\sin\theta)^2}{9} = 1$	M1	Substn. of $x = r\cos\theta$ and $y = r\sin\theta$ into given equn [$x = 2\cos\theta,\ y = 3\sin\theta$ is M0]
			so $r^2(9\cos^2\theta + 4\sin^2\theta) = 36$		
			$\Rightarrow r^2 = \frac{36}{9\cos^2\theta + 4\sin^2\theta}$	A1	or any equivalent expression where r^2 is the subject.
			$\Rightarrow r^2 = \frac{36\sec^2\theta}{9 + 4\tan^2\theta}$	A1	**AG**. mult. of num. and denom by $\sec^2\theta$ (or earlier division throughout by $\cos^2\theta$) needs to be explicitly seen.
				3	

1 (a) (iii)	Area $= \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{36\sec^2\theta}{9+4\tan^2\theta}\,d\theta$	B1	Correct statement (limits may be implied later)	
	$3u = 2\tan\theta \rightarrow 3du = 2\sec^2\theta\, d\theta$	B1	o.e. *(Allow use of u = 2tanθ etc)*	
	Area $= \left(\frac{1}{2}.36\right)\int \frac{3/2}{9+9u^2}\,du$	M1*	Attempt to use their dθ to express area in terms of u . Condone omission of $\frac{1}{2}$ and/ or 36 and limits.	
	$\Rightarrow$ Area = $27\int_{...}^{...}\frac{1}{9+9u^2}\,du = [3\arctan u]_{...}^{...}$	A1	award for $k\arctan u$. Ignore limits.	
	$\Rightarrow$ Area = $\left[3\arctan(\frac{2}{3}\tan\theta)\right]_{...}^{...}$	M1*	either k $\arctan\left(\frac{2}{3}\tan\theta\right)$ seen or attempt at finding both u limits. [$u = 0$ and $u = 2/3$]	
	Area = $3\left[\arctan\left(\frac{2}{3}\tan\frac{\pi}{4}\right) - \arctan(\frac{2}{3}\tan 0)\right]$	DM1	Attempt to substitute consistent limits into their expression. *Condone omission of –arctan0*	
	$= 3\arctan\left(\frac{2}{3}\right)$	A1 **7**	**AG**.	
1 (b)	$f(x) = \arctan(1+x)$			OR $g(x)$ = arctan x
	f(0) = arctan1 (= $\frac{\pi}{4}$)	B1		g(1) = arctan 1 (= $\pi/4$)
	$f'(x) = \frac{1}{1+(1+x)^2} = \frac{1}{x^2+2x+2}$	M1	Use of result in (a)(i) or equiv.	$g'(x) = 1/(1+x^2)$
	so $f'(0) = \frac{1}{2}$	A1	**CWO**	g'(1) = 1/2
	$f''(x) = \frac{-2(x+1)}{(x^2+2x+2)^2}$	A1	o.e.	$g''(x) = -2x/(1+x^2)^2$ g''(1) = −1/2
	so $f''(0) = -\frac{1}{2}$			
	so series begins $\frac{\pi}{4} + \frac{1}{2}x - \frac{1}{4}x^2$	A1 **5**	**CWO** Simplified (must be $\pi/4$ here)	

Question			Answer	Marks	Guidance
2	(a)		$C+\mathrm{j}S=-\frac{1}{2}e^{\mathrm{j}\theta}+\frac{1}{4}e^{2\mathrm{j}\theta}-\frac{1}{8}e^{3\mathrm{j}\theta}+\cdots$	M1	Forming C + jS as a series of powers
			G.P with $a=-\frac{1}{2}e^{\mathrm{j}\theta}$ and $r=-\frac{1}{2}e^{\mathrm{j}\theta}$	B1	For both soi
			Sum to infinity $=\dfrac{-\frac{1}{2}e^{\mathrm{j}\theta}}{1+\frac{1}{2}e^{\mathrm{j}\theta}}$	M1 A1	Attempt at forming expression for sum (to infinity). o.e.
			$=\dfrac{-\frac{1}{2}e^{\mathrm{j}\theta}}{1+\frac{1}{2}e^{\mathrm{j}\theta}}\times\dfrac{1+\frac{1}{2}e^{-\mathrm{j}\theta}}{1+\frac{1}{2}e^{-\mathrm{j}\theta}}$	DM1	Mult. num and denom. by $1+\frac{1}{2}e^{-\mathrm{j}\theta}$ o.e.
			$=\dfrac{-\frac{1}{2}e^{\mathrm{j}\theta}-\frac{1}{4}}{1+\frac{1}{4}+\frac{1}{2}e^{\mathrm{j}\theta}+\frac{1}{2}e^{-\mathrm{j}\theta}}$	DM1	Expanding numerator and denominator
			$=\dfrac{-\frac{1}{2}\cos\theta-\frac{1}{4}-\frac{1}{2}\mathrm{j}\sin\theta}{\frac{5}{4}+\cos\theta}$ $=\dfrac{-2\cos\theta-1-2\mathrm{j}\sin\theta}{5+4\cos\theta}$	DM1	Obtaining expression in trig functions, with a real denominator *(DM marks are dependent on all previous M marks)*
			$\Rightarrow S=\dfrac{-2\sin\theta}{5+4\cos\theta}$	A1	**AG**
			$C=\dfrac{-2\cos\theta-1}{5+4\cos\theta}$	A1 **9**	
2	(b)	(i)	Argand diagram showing point A in the first quadrant and point B in the fourth quadrant with A & B (approx) symmetrical about the real axis.	B1 **1**	*Allow z_1 and z_2 in place of A and B*

2	(b)	(ii)	**Either** $z_1 z_2 = (x + jy)(x - jy) = x^2 + y^2$ (which is real) so $z_1 z_2 = a^2$ **Or** $z_1 z_2 = ae^{j\theta}.ae^{-j\theta} = a^2$ (which is real) so $z_1 z_2 = a^2$ **Or** $\lvert z_1 z_2 \rvert = a^2$, $\arg(z_1 z_2) = \arg z_1 + \arg z_2 = 0$ so $z_1 z_2 = a^2$	**M1** **A1** Or **M1** **A1** Or **M1** **A1** **2**	Note: $z_1 z_2 = a^2$ WW is M0 as the lengths of side may have been multiplied without justification.
2	(b)	(iii)	$\gamma = \pi$	**B1** **1**	
2	(b)	(iv)	$\frac{z_3}{z_1} = 3e^{j\pi}$ or $3(\cos\pi + j\sin\pi)$ or [3, π] or mod = 3 arg = π	**B1** **B1** **2**	For modulus is 3 *(Just 3 is B0B0)* For argument is π (or $-\pi$) If B0B0, then **SC1** for $z_3 / z_1 = -3$
2	(b)	(v)	**Either** $\arg\left(\frac{z_1}{z_4}\right) = \arg z_1 - \arg z_4 = \frac{\pi}{6} - \frac{5\pi}{6} = \frac{-2\pi}{3}$ $\frac{z_1}{z_4} = \frac{1}{3}e^{-\frac{2}{3}j\pi}$ $\frac{z_1}{z_4} = -\frac{1}{6} - j\frac{\sqrt{3}}{6}$ **Or** $z_1 = a\left(\frac{\sqrt{3}}{2} + \frac{1}{2}j\right)$ and $z_4 = 3a\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}j\right)$ $\frac{z_1}{z_4} = -\frac{1}{6} - j\frac{\sqrt{3}}{6}$	**B1** **M1** **A1** Or **B1** **M1** **A1** **3**	an attempt at expressing *their* $\frac{z_1}{z_4}$ in polar form and converting to the form $x + jy$. (*x*, *y* both non-zero) Accept $-\frac{1}{6}(1 + j\sqrt{3})$ or exact equivalents. for both. an attempt at expressing *their* $\frac{z_1}{z_4}$ in the form $x + jy$. (with z_4 not real or pure imaginary) Accept $-\frac{1}{6}(1 + j\sqrt{3})$ or exact equivalents.

3	(a)	(i)	det **M** = $k(2-4)-2(-6-2)+1(6+1)$ det **M** = 0 $\Rightarrow -2k+16+7=0$ $\Rightarrow k=\frac{23}{2}$	M1 DM1 A1 **3**	Attempt at determinant; condone one error. Setting determinant to zero and attempt at solution.
3	(a)	(ii)	det **M** = 23 $-2k$ $\mathbf{M}^{-1}=\frac{1}{23-2k}\begin{pmatrix}-2 & 6 & 5\\ 8 & -2k-1 & 3-2k\\ 7 & 2-2k & -k-6\end{pmatrix}$	M1 A1 DM1 A1 **4**	At least 4 cofactors correct. M0 if multiplied by the corresponding element. 6 cofactors correct. Transposing and multiplying by 1/det M cao.
3	(b)	(i)	det $(\mathbf{Q}-\lambda\mathbf{I})=0$ $\Rightarrow$ det $\begin{pmatrix}3-\lambda & 3\\ 4 & 7-\lambda\end{pmatrix}=0$ $\Rightarrow (3-\lambda)(7-\lambda)-12=0$ $\Rightarrow \lambda^2-10\lambda+9=0$ so $\lambda=1,9$ For $\lambda=1$, $\begin{pmatrix}2 & 3\\ 4 & 6\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\mathbf{0}$ $\Rightarrow 2x+3y=0$ so eigenvector is $\begin{pmatrix}3\\ -2\end{pmatrix}$ o.e For $\lambda=9$, $\begin{pmatrix}-6 & 3\\ 4 & -2\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\mathbf{0}$ $\Rightarrow 2x-y=0$ so eigenvector is $\begin{pmatrix}1\\ 2\end{pmatrix}$ o.e	 M1 A1 M1 A1 A1 **5**	 Forming characteristic equation. For either λ, finding eqn in x and y from $(\mathbf{Q}-\lambda\mathbf{I})\begin{pmatrix}x\\ y\end{pmatrix}=\mathbf{0}$ o.e.

3	(b)	(ii)	$\mathbf{P} = \begin{pmatrix} 3 & 1 \\ -2 & 2 \end{pmatrix}$ $\mathbf{D} = \begin{pmatrix} 1 & 0 \\ 0 & 9 \end{pmatrix}$	**B1ft** **B1ft** **2**	For B2, columns must be consistent
3	(b)	(iii)	$\mathbf{P}^{-1} = \frac{1}{8}\begin{pmatrix} 2 & -1 \\ 2 & 3 \end{pmatrix}$ $\mathbf{Q}^n = \mathbf{P}\mathbf{D}^n\mathbf{P}^{-1}$ $= \frac{1}{8}\begin{pmatrix} 3 & 1 \\ -2 & 2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 9^n \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 2 & 3 \end{pmatrix}$ $\mathbf{Q}^n = \frac{1}{8}\begin{pmatrix} 3 & 9^n \\ -2 & 2\times 9^n \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 2 & 3 \end{pmatrix}$ $= \frac{1}{8}\begin{pmatrix} 6+2\phi & 3\phi-3 \\ 4\phi-4 & 6\phi+2 \end{pmatrix}$ where $\phi = 9^n$	**B1ft** **M1** **DM1** **A1** **4**	(ft provided their **P** has an inverse) Forming product Both matrix products attempted **AG** *By induction: M2A1 for inductive step* *A1 for checking n = 1 and completion*

Question		Answer	Marks	Guidance
4	(i)	**Either** $\text{sech}^2 x + \tanh^2 x =$ $\frac{4}{(e^x+e^{-x})^2} + \frac{(e^x-e^{-x})^2}{(e^x+e^{-x})^2}$ $= \frac{e^{2x}+2+e^{-2x}}{(e^x+e^{-x})^2}$ $= \frac{(e^x+e^{-x})^2}{(e^x+e^{-x})^2}$ = 1 **Or** $\cosh^2 x - \sinh^2 x =$ $\frac{(e^x+e^{-x})^2}{4} - \frac{(e^x-e^{-x})^2}{4}$ $= \frac{e^{2x}+2+e^{-2x} - e^{2x}+2-e^{-2x}}{4}$ $= 1$	**M1** **A1** **DM1** **A1** Or **M1** **A1** **DM1** **A1** **4**	Use of $\frac{1}{\cosh^2 x}$ and $\frac{\sinh^2 x}{\cosh^2 x}$ Combining and expanding numerator (allow one error) Showing identity is equiv to $\cosh^2 x - \sinh^2 x = 1$ Use of definitions of cosh*x* and sinh*x* in $\cosh^2 x - \sinh^2 x$ Expanding (allow one error) Completion
4	(ii)	Let $\tanh y = x$ $\Rightarrow \frac{e^y - e^{-y}}{e^y + e^{-y}} = x$ $\Rightarrow e^y - e^{-y} = x(e^y + e^{-y})$ $\Rightarrow e^{2y} - 1 = x(e^{2y}+1)$ $\Rightarrow e^{2y} = \frac{1+x}{1-x}$ $\therefore \text{artanh}x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$ valid if $\|x\| < 1$	**M1** **DM1** **A1** **A1** **B1** **5**	$y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ is M0 unless *x*, *y* interchanged later Attempt to find e^{2y} in terms of *x*. e^{2y} in terms of *x* **AG** Strict (e.g. $-1 \le x < 1$ is B0)

4	(iii)	$3(\tanh^2 x-(1-\tanh^2 x))=\tanh x-2$ $\Rightarrow 6\tanh^2 x-\tanh x-1=0$ $\Rightarrow (3\tanh x+1)(2\tanh x-1)=0$ $\Rightarrow \tanh x=\frac{1}{2}, \tanh x=-\frac{1}{3}$ $\text{artanh}a=\frac{1}{2}\ln\left(\frac{1+a}{1-a}\right)$ $\therefore x=\frac{1}{2}\ln 3, x=-\frac{1}{2}\ln 2$ o.e. Or Write in terms of exponentials $2e^{4x}-7e^{2x}+3=0$ $e^{2x}=3,\ ½$ $x=½\ln 3,\quad -½\ln 2$	M1 DM1 A1 DM1 A1 M2 DM1 A1 A1 5	Use of identity from **(i)** Solving quadratic in tanh*x* For both. Using result from part (ii) at least once. For both. Accept ½ ln ½ , $\ln\sqrt{3}$ etc Obtaining three term quadratic eqn for e^{2x} Solving to obtain e^{2x} For both For both
4	(iv)	$\int \frac{1}{\tanh x-\text{sech}x}\,dx=\int\frac{\cosh x}{\sinh x-1}\,dx$ $=\ln\lvert\sinh x-1\rvert\ (+c)$ $=\ln\lvert\sinh(\text{arsinh}3)-1\rvert-\ln\lvert\sinh(\text{arsinh}2)-1\rvert$ $=\ \ln 2$	M1 A1 DM1 A1 4	Ignore limits. OR $\int\frac{e^x+e^{-x}}{e^x-e^{-x}-2}\,dx$ (Modulus not needed) $=\ln(e^x-e^{-x}-2)$ Substituting consistent limits into a logarithmic function.

	Alternative for 4(ii)		
	If $y = \text{artanh } x,\ \frac{dy}{dx} = \frac{1}{\text{sech}^2 y} = \frac{1}{1-x^2}$ $y = \int \frac{1}{2}\left(\frac{1}{1+x} + \frac{1}{1-x}\right) dx$ $y = \frac{1}{2}(\ln(1+x) - \ln(1-x))\quad (+c)$ $\therefore\ \text{artanh} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$	**M1** **M1** **A1** **A1** **4**	 Dependent on at least one M1 Completion, including showing that *c* = 0 (Dependent on M2A1)

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4756 Further Methods for Advanced Mathematics (FP2)

General Comments:

Most candidates were able to demonstrate a good understanding of the topics being examined. The marks were generally high with a fair number achieving full marks. Q.1 (on calculus) and Q.3 (on matrices) were answered somewhat better than Q.2 (on complex numbers) and Q.4 (on hyperbolic functions).

Comments on Individual Questions:

Q.1(a)(i) Most candidates differentiated the equation correctly. Relating $\mathrm{d}y/\mathrm{d}x$ to $1/(a^2 + x^2)$ caused some difficulty, and the factor a was often missing.

Q.1(a)(ii) Most candidates substituted $x = r\cos\theta$ and $y = r\sin\theta$ and obtained $r^2 = 36/(9\cos^2\theta + 4\sin^2\theta)$. However, many then just put this equal to the given answer without explanation, and this lost a mark. As the answer is given on the question paper, each step should be fully justified.

Q.1(a)(iii) Most candidates wrote down a correct integral expression for the required area; the factor ½ was sometimes omitted, and a square root sometimes introduced. The procedure for integration by substitution was well understood and the given result was very often obtained convincingly.

Q.1(b) Candidates who differentiated $f(x)$ twice usually obtained the correct series. Many started by replacing x with $(1 + x)$ in the standard series for $\arctan(x)$, but this did not achieve anything useful.

Q.2(a) Almost all candidates recognised that $C + \mathrm{j}S$ was a geometric series and could write down its sum to infinity. This earned 4 marks out of the 9, however many candidates made no progress beyond this. Those who understood how to realise the denominator were usually able to complete this part.

Q.2(b)(i) Most of the Argand diagrams were correct. Reasons for not gaining full credit included not labelling the points, or showing symmetry in the imaginary axis instead of the real axis.

Q.2(b)(ii) This was generally well done, using either the polar form or the Cartesian form for z_1 and z_2. Some unfortunately used $a \pm \mathrm{j}b$ and then became confused over the meaning of a.

Q.2(b)(iii) The value of γ was usually given correctly.

Q.2(b)(iv) The argument was usually given correctly, but the modulus was quite often given as 1 or 1/3 instead of 3. This part was omitted by about 10% of the candidates.

Q.2(b)(v) This part caused quite a lot of difficulty, and was omitted by about 15% of candidates. It was necessary to use the fact that triangle OAB was equilateral to obtain the argument of z_1 as $\pi/6$, but many candidates gave $\pi/3$ instead, and some tried to answer the question without finding a value for the argument of z_1.

Q.3(a)(i) Almost all candidates successfully found the value of k making the determinant equal to zero.

Q.3(a)(ii) The method for finding the inverse of a 3 x 3 matrix was very well understood, although there were quite a few careless slips.

Q.3(b)(i) Almost all candidates knew how to find the eigenvalues and corresponding eigenvectors, and the work was usually completed accurately.

Q.3(b)(ii) Almost all candidates understood how **P** and **D** are related to the eigenvectors and eigenvalues.

Q.3(b)(iii) Most candidates understood how to find $\mathbf{Q}^n$, and the given result was usually obtained convincingly.

Q.4(i) This was generally well answered.

Q.4(ii) The logarithmic form for artanh(x) was very often derived confidently by converting $x = \tanh(y)$ into exponential form. Some candidates used the quadratic formula to obtain e^y from the equation $(1 - x)e^{2y} - (1 + x) = 0$ resulting in unnecessarily complicated working. The condition $|x| < 1$ was quite often omitted, or given wrongly, despite being printed in the formula book.

Q.4(iii) The usual approach was to use the identity in part (i) to form a quadratic equation for tanh(x), then use the result of part (ii) to convert the two values of x into logarithmic form. This was very often completed accurately. Another approach was to convert the equation into exponential form, which gave a quadratic equation for e^{2x}; but this tended to be less successful.

Q.4(iv) About 10% of the candidates omitted this part, and many others could not make significant progress. Those who wrote the expression to be integrated as $\cosh(x)/(\sinh(x) - 1)$ usually carried on to obtain the correct answer.

Unit level raw mark and UMS grade boundaries June 2017 series

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AS GCE / Advanced GCE / AS GCE Double Award / Advanced GCE Double Award

GCE Mathematics (MEI)											
				Max Mark	a	b	c	d	e	u
4751	01	C1 – MEI Introduction to advanced mathematics (AS)	Raw	72	63	58	53	49	45	0
			UMS	100	80	70	60	50	40	0
4752	01	C2 – MEI Concepts for advanced mathematics (AS)	Raw	72	55	49	44	39	34	0
			UMS	100	80	70	60	50	40	0
4753	01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	54	49	45	41	36	0
4753	02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753	82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4754	01	C4 – MEI Applications of advanced mathematics (A2)	Raw	90	67	61	55	49	43	0
			UMS	100	80	70	60	50	40	0
4755	01	FP1 – MEI Further concepts for advanced mathematics (AS)	Raw	72	57	52	47	42	38	0
			UMS	100	80	70	60	50	40	0
4756	01	FP2 – MEI Further methods for advanced mathematics (A2)	Raw	72	65	58	52	46	40	0
			UMS	100	80	70	60	50	40	0
4757	01	FP3 – MEI Further applications of advanced mathematics (A2)	Raw	72	64	56	48	41	34	0
			UMS	100	80	70	60	50	40	0
4758	01	(DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	63	56	50	44	37	0
4758	02	(DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758	82	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4761	01	M1 – MEI Mechanics 1 (AS)	Raw	72	57	49	41	34	27	0
			UMS	100	80	70	60	50	40	0
4762	01	M2 – MEI Mechanics 2 (A2)	Raw	72	56	48	41	34	27	0
			UMS	100	80	70	60	50	40	0
4763	01	M3 – MEI Mechanics 3 (A2)	Raw	72	58	50	43	36	29	0
			UMS	100	80	70	60	50	40	0
4764	01	M4 – MEI Mechanics 4 (A2)	Raw	72	53	45	38	31	24	0
			UMS	100	80	70	60	50	40	0
4766	01	S1 – MEI Statistics 1 (AS)	Raw	72	61	55	49	43	37	0
			UMS	100	80	70	60	50	40	0
4767	01	S2 – MEI Statistics 2 (A2)	Raw	72	56	50	45	40	35	0
			UMS	100	80	70	60	50	40	0
4768	01	S3 – MEI Statistics 3 (A2)	Raw	72	63	57	51	46	41	0
			UMS	100	80	70	60	50	40	0
4769	01	S4 – MEI Statistics 4 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4771	01	D1 – MEI Decision mathematics 1 (AS)	Raw	72	52	46	41	36	31	0
			UMS	100	80	70	60	50	40	0
4772	01	D2 – MEI Decision mathematics 2 (A2)	Raw	72	53	48	43	39	35	0
			UMS	100	80	70	60	50	40	0
4773	01	DC – MEI Decision mathematics computation (A2)	Raw	72	46	40	34	29	24	0
			UMS	100	80	70	60	50	40	0
4776	01	(NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	58	53	48	43	37	0
4776	02	(NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776	82	(NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
4777	01	NC – MEI Numerical computation (A2)	Raw	72	55	48	41	34	27	0

		UMS	100	80	70	60	50	40	0
4798	01 FPT - Further pure mathematics with technology (A2)	Raw	72	57	49	41	33	26	0
		UMS	100	80	70	60	50	40	0

GCE Statistics (MEI)

			Max Mark	a	b	c	d	e	u
G241	01 Statistics 1 MEI (Z1)	Raw	72	61	55	49	43	37	0
		UMS	100	80	70	60	50	40	0
G242	01 Statistics 2 MEI (Z2)	Raw	72	55	48	41	34	27	0
		UMS	100	80	70	60	50	40	0
G243	01 Statistics 3 MEI (Z3)	Raw	72	56	48	41	34	27	0
		UMS	100	80	70	60	50	40	0

GCE Quantitative Methods (MEI)

			Max Mark	a	b	c	d	e	u
G244	01 Introduction to Quantitative Methods MEI	Raw	72	58	50	43	36	28	0
G244	02 Introduction to Quantitative Methods MEI	Raw	18	14	12	10	8	7	0
		UMS	100	80	70	60	50	40	0
G245	01 Statistics 1 MEI	Raw	72	61	55	49	43	37	0
		UMS	100	80	70	60	50	40	0
G246	01 Decision 1 MEI	Raw	72	52	46	41	36	31	0
		UMS	100	80	70	60	50	40	0

Level 3 Certificate and FSMQ raw mark grade boundaries June 2017 series

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Level 3 Certificate Mathematics for Engineering

				Max Mark	a*	a	b	c	d	e	u
H860	01	Mathematics for Engineering		This unit has no entries in June 2017							
H860	02	Mathematics for Engineering									

Level 3 Certificate Mathematical Techniques and Applications for Engineers

				Max Mark	a*	a	b	c	d	e	u
H865	01	Component 1	Raw	60	48	42	36	30	24	18	0

Level 3 Certificate Mathematics - Quantitative Reasoning (MEI) (GQ Reform)

				Max Mark	a	b	c	d	e	u
H866	01	Introduction to quantitative reasoning	Raw	72	54	47	40	34	28	0
H866	02	Critical maths	Raw	60*	48	42	36	30	24	0
		*Component 02 is weighted to give marks out of 72	Overall	144	112	97	83	70	57	0

Level 3 Certificate Mathematics - Quantitive Problem Solving (MEI) (GQ Reform)

				Max Mark	a	b	c	d	e	u
H867	01	Introduction to quantitative reasoning	Raw	72	54	47	40	34	28	0
H867	02	Statistical problem solving	Raw	60*	41	36	31	27	23	0
		*Component 02 is weighted to give marks out of 72	Overall	144	103	90	77	66	56	0

Advanced Free Standing Mathematics Qualification (FSMQ)

				Max Mark	a	b	c	d	e	u
6993	01	Additional Mathematics	Raw	100	72	63	55	47	39	0

Intermediate Free Standing Mathematics Qualification (FSMQ)

				Max Mark	a	b	c	d	e	u
6989	01	Foundations of Advanced Mathematics (MEI)	Raw	40	35	30	25	20	16	0